

## Problems

1. Students measuring the dimensions of a table top use a meter stick. They determine that the width of the table is between 78.4 cm and 78.3 cm.

2. Express the measurement and uncertainty in the form:  $x \pm \Delta x$ . ~~78.35~~  $78.35 \pm 0.05$  cm

3. What is the absolute uncertainty of the width measurement?  $0.05$  cm

4. What is the relative uncertainty of the width measurement?  $\frac{0.05 \text{ cm}}{78.35 \text{ cm}} = 0.064\%$

5. Using the same meter stick to measure the thickness of the table, the students determine that the thickness is between 3.5 cm and 3.6 cm.

6. Express the measurement and uncertainty in the form:  $x \pm \Delta x$ .  $3.55 \pm 0.05$  cm

7. What is the absolute uncertainty of the thickness measurement?  $0.05$  cm

8. What is the relative uncertainty of the thickness measurement?  $\frac{0.05}{3.55} = 1.4\%$

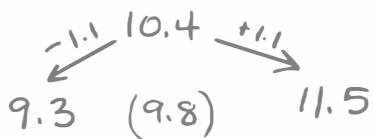
9. Compare the relative uncertainties of the width and thickness. Why are they so different if the same meter stick was used for each measurement?

width: absolute uncertain very small compare to measurement

thickness: absolute uncertainty comparable to measurement

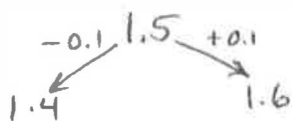
10. Consider the following results for different experiments. Determine if they agree with the accepted or predicted result listed to the right. Also calculate the percent difference for each result.

a) measured value for  $g = 10.4 \pm 1.1 \text{ m/s}^2$  (accepted value for  $g = 9.8 \text{ m/s}^2$ )



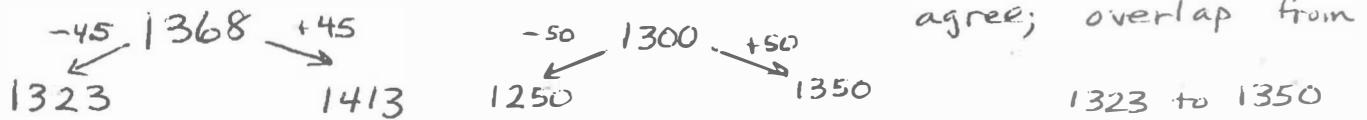
agree with predicted;  $\% \text{ diff} = \frac{|10.4 - 9.8|}{9.8} = 6.1\%$

b) measured value for  $T = 1.5 \pm 0.1$  sec (predicted value for  $T = 1.1$  sec)



does not agree;  $\% \text{ diff} = \frac{|1.5 - 1.1|}{1.1} = 36.4\%$

c) measured value for  $k = 1368 \pm 45 \text{ N/m}$  (predicted value for  $k = 1300 \pm 50 \text{ N/m}$ )



11. Each member of your lab group weighs an empty box and two metal bars twice. The following table shows this data.

trial	Box (g)	deviation	Bar 1 (g)	deviation	Bar 2 (g)	deviation
1	201.3	0.13	98.7	0.62	95.6	0.22
2	201.5	0.33	98.8	0.72	95.3	0.52
3	202.3	1.13	96.9	1.18	96.4	0.38
4	202.1	0.93	97.1	0.98	96.2	0.38
5	199.8	1.37	98.4	0.32	95.8	0.02
6	200.0	1.17	98.6	0.52	95.6	0.22
average	201.17	$\pm 0.84$	98.08	$\pm 0.72$	95.82	$\pm 0.29$

$\uparrow$  avg. dev       $\uparrow$  avg. dev       $\uparrow$  avg. dev

a. Estimate the uncertainty of each data set by finding the average deviations.

b. Calculate the total mass of the box with Bar 1. Use rules for uncertainty propagation.

$$(201.17 \pm 0.84) + (98.08 \pm 0.72) = 299.25 \pm 1.56 \text{ g}$$

c. Calculate the mass of the box with Bar 2. Use rules for uncertainty propagation.

$$(201.17 \pm 0.84) + (95.82 \pm 0.29) = 296.99 \pm 1.13 \text{ g}$$

d. Calculate the mass of the box with both bars. Use rules for uncertainty propagation.

$$(201.17 \pm 0.84) + (98.08 \pm 0.72) + (95.82 \pm 0.29) = 395.07 \pm 1.85 \text{ g}$$

12. The area of a rectangular metal plate was found by measuring its length and its width. The length was found to be  $5.37 \pm 0.05 \text{ cm}$ . The width was found to be  $3.42 \pm 0.02 \text{ cm}$ .

a. What are the relative uncertainties of each measurement?

$$\frac{0.05}{5.37} = 0.93\% \quad \frac{0.02}{3.42} = 0.58\%$$

Add rel. uncert

$1.51\%$

What is the area, including the uncertainty? (Use the method of adding relative uncertainties.)

$$5.37 \times 3.42 = 18.37$$

$$1.51\% \times 18.4 = 0.27$$

$\text{Area} = 18.37 \pm 0.27 \text{ cm}^2$

Add measurements and add uncertainty

## Discussion Questions

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1. How is the word *uncertainty* used differently in everyday speech than in science?
2. Does a greater degree of uncertainty affect your confidence in the results?
3. A scientist makes a prediction and claims that they are completely certain of the outcome. How does this affect your confidence in the outcome?
4. What is the difference between uncertainty and error?
5. Students just starting science often attribute results that they think are incorrect to "human error". More advanced science students recognize that this is not a sufficient description of potential problems in lab work. Why?
6. What is the difference between the scientific use of the word *uncertainty* and the everyday use?
7. Does the knowledge that the results of a scientific prediction have uncertainty increase or decrease your confidence in the prediction?
8. What would be your reaction to a scientific prediction that is 100% certain, that is, a prediction that has no uncertainty?
9. You are measuring the time it takes for a student to run a 100-meter race. Describe a method you could use to determine the uncertainty of the time.
10. What does it mean to be absolutely certain? What things can we be absolutely certain about?

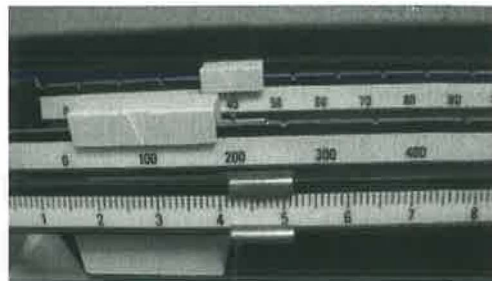
Uncertainty is thinking about measurement as a range, rather than a number. By  $\pm$  the uncertainty of the measurement, we actually get a "certainty" range for our measurement. The smaller the "uncertainty" the more certain we are of the measurement.

### Sample Quiz Questions

1. Students are trying to identify an unknown liquid by determining its density and comparing it to a table of densities of known liquids. They begin by finding the mass of a graduated cylinder, which they determine to be  $54.55 \pm 0.05$  grams. What is the relative uncertainty of this measurement?

$$\frac{0.05}{54.55} = 0.092\%$$

2. The scale at right shows the mass of the graduated cylinder from problem 2 filled with some of the unknown liquid. Determine the reading on the beam balance at right, including absolute uncertainty. What is the relative uncertainty of the measurement?



$$144.61 \pm 0.05 \text{ g so...}$$

$$\frac{0.05}{144.61} = 0.035\%$$

3. What is the mass of the liquid in the graduated cylinder, including uncertainty? What is the relative uncertainty of this measurement?

$$(144.61 \pm 0.05) - (54.55 \pm 0.05) = 90.06 \pm 0.10 \text{ g}$$

$$(\text{relative uncertainty} = 0.11\%)$$

4. By reading the graduated cylinder, the students determine that the volume of liquid is  $114 \pm 2$  ml. What is the density of the unknown liquid, including uncertainty? (note: use the method of adding relative uncertainties)

$$\text{relative} = \frac{2}{114} = 1.8\%$$

Note: Add relative uncert. convert to absolute uncert.

$$D = \frac{M}{V} = \frac{90.06 \pm 0.11\%}{114 \pm 1.8\%} = 0.790 \pm 1.91\% = 0.79 \pm 0.015 \text{ g/mL}$$

(1.91% of 0.79 = 0.015)

5. Shown at right is a table of densities of various alcohols. What conclusions can the students reach about the identity of the unknown liquid based on this table and the results of their density calculations?

Compound	Density (g/ml)
Methanol	0.791
Ethanol	0.789
Isopropanol	0.785

$$0.775 \leftarrow 0.790 \rightarrow 0.805$$

The liquid could be any of the alcohols listed.

6. Identify one plausible source of systematic error in this procedure and describe how to correct it.

The triple beam balance might not have been properly "zeroed" before use. Also, liquid in the graduated cylinder before use.

7. Identify one source of random error in this procedure and describe how to correct it.

Determining when the triple beam was balanced enough to read.